# Cryptanalysis of Low-Data Instances of Full LowMCv2

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# Outline


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- LowMC Description
- Related Work

New Technique
- Overview of the Technique
- Proposed Framework

Key Recovery
- Simplified Representation of LowMC
- Impact on Applications of LowMC

Conclusion

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- Examples of such designs include LowMC, Kreyvium, Flip, MiMC and Rasta.

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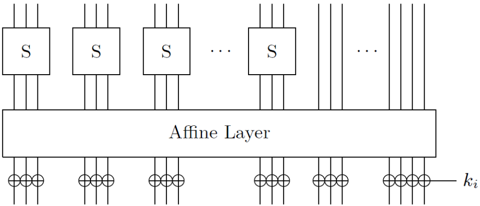

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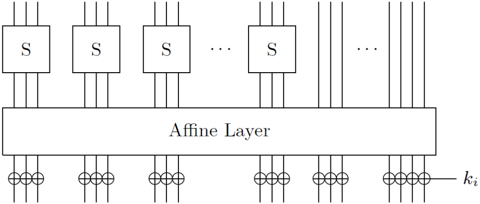

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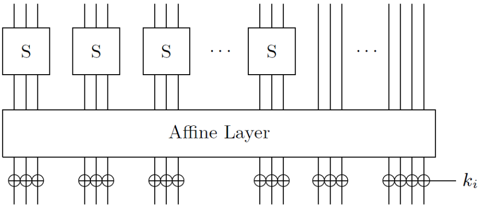

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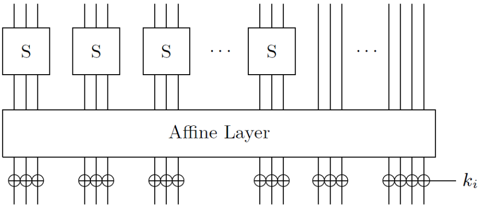
  - Round key is generated by a randomly chosen multiplication of a full-rank $b \times k$ with the master key.



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LowMCv3 is used in all applications we are aware of, e.g Picnic signature scheme (Zaverucha et al., CCS 2017), group signature schemes (Boneh et al., Derler et al.), or a protype Signal 'plugin' for private contact discovery.

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## This Work

Exploit previous ideas to take advantage of the positive properties and overcome the limitations!

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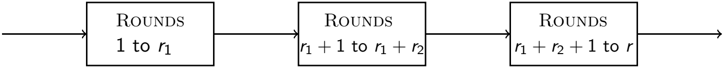


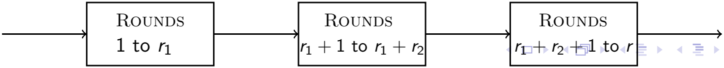

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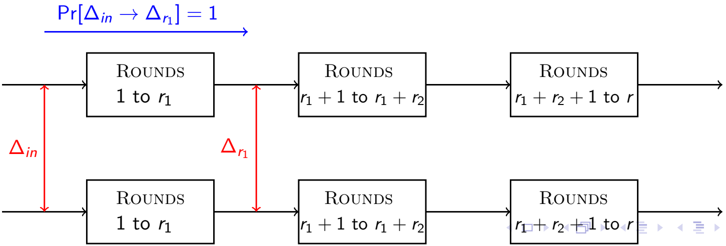

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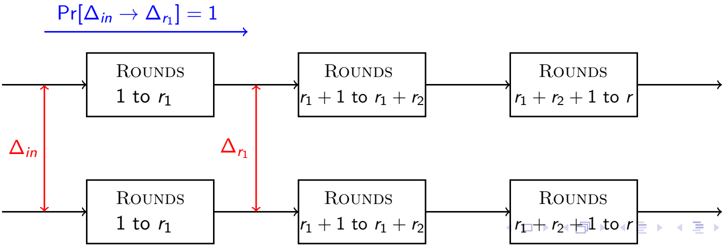

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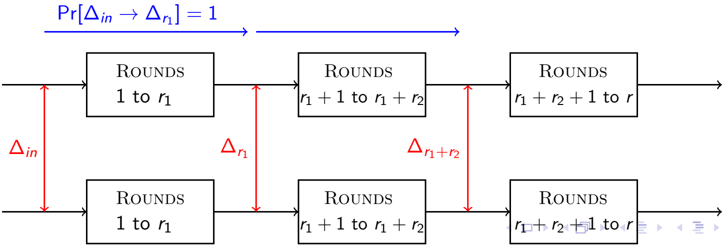

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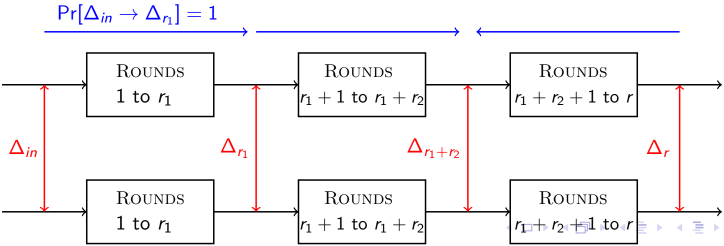

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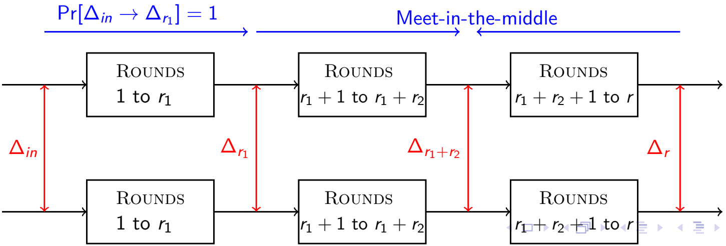

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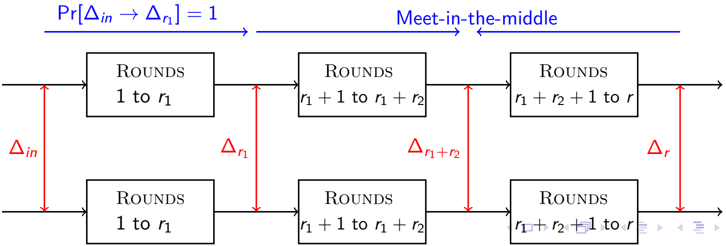
- Repeat the procedure to find all intermediate differences.



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- We can cover $r_1 = \left\lceil \frac{b}{3 \cdot m} \right\rceil - 1$ rounds.

# Estimating the Number of Reachable Differences

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- How can we overcome this limitation?

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### Condition to Avoid Wrong Collision

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# Key Recovery

### Definition

An Sbox $S : \{0,1\}^n \to \{0,1\}^n$ is called to be differentially $\delta$-uniform if for any $(\alpha, \beta) \in (\mathbb{F}_2^n \times \mathbb{F}_2^n)$, we have:

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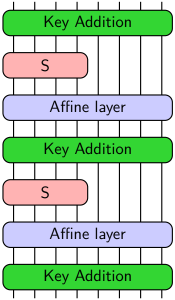
## Key candidates

We expect to have at most $2^{m.x}$ solutions for the quadratic $(X_r^I, X_r'^I, X_r^S, , X_r'^S)$, since each Sbox is differentially $2^x$-uniform. Each solution uniquely suggests a candidate for the round key $sk_r$ as follows:

$$C \oplus sk_r = X_r^L = \mathcal{L}(X_r^S) \rightarrow sk_r = C \oplus \mathcal{L}(X_r^S)$$

# Equivalent representation with Equivalent Round Keys




$\Rightarrow$

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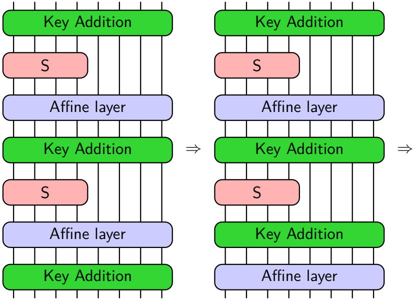

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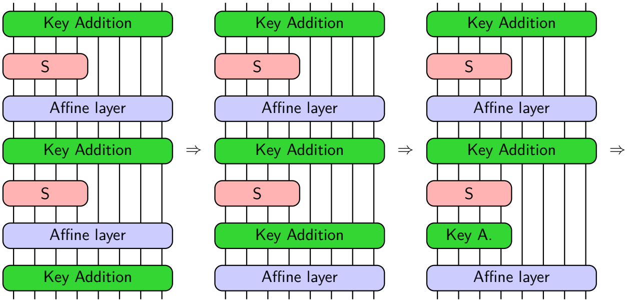

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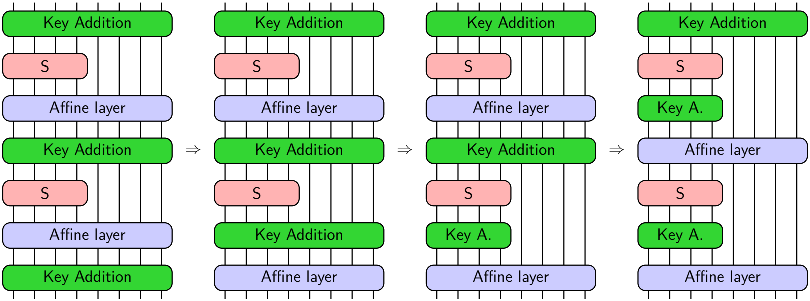

# Results

| Cipher Specification | | | | | Attack Details | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Block $n$ | S-boxes $m$ | Data $D$ | Key $k$ | Rounds $r$ | Dimension $d$ | $r_0$ $\lfloor\frac{n-\log_2 d}{3\cdot m}\rfloor$ | $r_1$ $\lfloor\frac{r-r_0}{2}\rfloor$ | $r_2$ $\lceil\frac{r-r_0}{2}\rceil$ | Time Complexity $2\cdot(\delta_d^{r_1}+\delta_d^{r_2})$ | Data $2(d+1)$ |
| 128 | 1 | 16 | 256 | 158 | 4 | 41 | 58 | 58 | $2^{164.9}$ | 10 |
| 128 | 5 | 16 | 256 | 37 | 4 | 8 | 14 | 15 | $2^{212.75}$ | 10 |
| 256 | 1 | 8 | 256 | 243 | 2 | 85 | 79 | 79 | $2^{223}$ | 6 |
| 256 | 5 | 8 | 256 | 53 | 2 | 17 | 18 | 18 | $2^{254.9}$ | 6 |
| 512 | 1 | 8 | 256 | 413 | 1 | 170 | 121 | 121 | $2^{226.6}$ | 4 |
| 1024 | 1 | 8 | 512 | 758 | 1 | 341 | 208 | 209 | $2^{389.9}$ | 4 |

- Several low-data instances of LowMCv2 can be broken significantly faster than exhaustive search.
- The type of instance that is vulnerable (few Sboxes per round) are used e.g. in post-quantum signature schemes.

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- A new insight into the security evaluation of block ciphers with a partial non-linear layer by presenting a new cryptanalytic technique.

# Conclusions

- New representation for the block ciphers with partial non-linear layer.
- A new insight into the security evaluation of block ciphers with a partial non-linear layer by presenting a new cryptanalytic technique.
- Best results for some versions of LowMC. Led to a new round 'formula' v3.