CRYPTANALYSIS OF AES-PRF AND ITS DUAL

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Overview

1. Background and Motivation

2. Preliminary

3. Overview of Our Attacks

4. Attacks on AES-PRF

5. Attacks on Dual-AES-PRF

6. Summary and Conclusion
BACKGROUND AND MOTIVATION
Background

Pseudorandom permutation (PRP)

- Main primitives in symmetric-key cryptography
- Ultimate security goal in the design of block ciphers
- Many secure block ciphers are readily available, e.g., AES
BACKGROUND

Pseudorandom permutation (PRP)

▶ Main primitives in symmetric-key cryptography
▶ Ultimate security goal in the design of block ciphers
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Pseudorandom function (PRF)

▶ Invertibility is unnecessary
▶ CTR encryption mode, authenticated encryption GCM
BACKGROUND

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PRP-to-PRF conversion

- Large efficiency costs design, e.g., Truncation, XOR of Permutations (XoP), Encrypted Davies-Meyer (EDM), The Dual of EDM (EDMD)
- Dedicated design with small efficiency costs, e.g., FastPRF,

\[ \text{FastPRF}_K(X) = E_K(X) \oplus E_K^1(X). \]
Motivation

Observations

- $\text{AES-PRF}_{s,t}$ is as efficient as AES
- Efficiency and cost-effectiveness comes at the cost of provable security
- Provable security result of EDMD no longer applies to AES-PRF

Open Problems

- $(s, t) = (2, 8)$ is left as an open question
- The security of $\text{AES-PRF}_{s,t}$
- The security of the dual version ($\text{Dual-AES-PRF}$)

Methods

- ID, ZC, DC, and MITM
Preliminary

Preliminary
AES-PRF & Dual-AES-PRF

- **AES-PRF<sub>s,t</sub>** (Mennink and Neves @ FSE 2018)

- **Dual-AES-PRF<sub>s,t</sub>**
Overview of Our Attacks
**Attacks on AES-PRF**

**Impossible differential/Zero-correlation attacks (s ≤ 2)**

![Diagram of impossible differential/zero-correlation attacks]

**Zero-correlation distinguishers (t ≤ 4)**

![Diagram of zero-correlation distinguishers]

**Meet-in-the-middle attacks on AES-PRF_{s,7−s}**
Attacks on Dual-AES-PRF

Impossible differential/Zero-correlation attacks ($t \leq 2$)

Differential attacks ($s \leq 4$)
ATTACKS ON AES-PRF
Impossible Differential Attack for AES-PRF

\[ s \text{ rounds} \rightarrow \Delta_{\text{in}}/\Gamma_{\text{in}} \rightarrow \Delta_{\text{out}}/\Gamma_{\text{out}} \rightarrow \Delta_{\text{out}}/\Gamma_{\text{out}} \rightarrow 0 \rightarrow \Delta_{\text{out}}/\Gamma_{\text{out}} \rightarrow \text{contradiction} \rightarrow \text{key recovery} \]

Diagram:
- \( K_0 \) and \( K_1 \)
- \( P \), \( x_1 \), \( x_1^i \), \( x_1^r \)
- \( SB \), \( SR \)
- \( MC \), \( MC^{-1} \)
- \( F \)
- \( x_2 \), \( x_2^i \), \( x_2^r \)
- \( MC^{-1}(K_2) \)
- \( C \)
Zero-Correlation Linear Attack for AES-PRF$_{2,8}$

Motivation  Preliminary  Overview  Attacks on AES-PRF  Attacks on Dual-AES-PRF  Conclusion
Distinguishers against AES-PRF$_{7,3}$ & AES-PRF$_{6,4}$

ZC Distinguisher for AES$_3$

ZC Distinguisher for AES$_4$
**Attack against AES-PRF\textsubscript{3,4}**

The number of possible sequences: $\left(2^8\right)^{255} = 2^{2040} \rightarrow \left(2^8\right)^{25} = 2^{200}$
Attacks on Dual-AES-PRF
Impossible Differential Attack for Dual-AES-PRF
t rounds

contradiction

key recovery
**Zero-correlation attack for Dual-AES-PRF**

\[
\Delta_{\text{in}} \quad \Gamma_{\text{in}} \quad \Delta_{\text{in}} \quad \Gamma_{\text{in}} \quad 0 \quad \Delta_{\text{in}} \quad \Gamma_{\text{in}} \quad t \text{ rounds} \quad \Delta_{\text{out}} \quad \Gamma_{\text{out}}
\]

Contradiction → Key recovery

\[
P \xrightarrow{S R^{-1}} 8 \text{ rounds} \xrightarrow{S R} K_{10}
\]

\[
SB^{-1}(x_9^R) \xrightarrow{SB} x_9^R \xrightarrow{MC} x_9^E \xrightarrow{MC^{-1}} MC^{-1}(K_9) \xrightarrow{SB} x_9 S \xrightarrow{SR} x_{10}^R \xrightarrow{SR^{-1}} C
\]
Differential Attack for Dual-AES-PRF$^4,6$

\[ \Delta_{in} \rightarrow s \text{ rounds} \rightarrow \Delta_{in} \rightarrow \oplus \rightarrow 0 \rightarrow \ast \rightarrow 0 \]

key recovery

\[ K_0 \rightarrow P \rightarrow x_1^P \rightarrow S_{B_1} \rightarrow S_{R_1} \rightarrow E_{K_1} \rightarrow x_1^E \rightarrow K_1 \]

\[ \Delta_{in} \rightarrow 6 \text{ rounds} \rightarrow C \}

\[ EK_2 \rightarrow x_2^E \rightarrow S_{B_2} \rightarrow S_{R_2} \rightarrow x_2^S \rightarrow P \]

\[ EK_3 \rightarrow x_3^E \rightarrow S_{B_3} \rightarrow S_{R_3} \rightarrow x_3^S \rightarrow P \]

\[ EK_4 \rightarrow x_4^E \rightarrow S_{B_4} \rightarrow S_{R_4} \rightarrow x_4^S \rightarrow P \]
Summary and Conclusion
## Summary

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<th>Target</th>
<th>s</th>
<th>t</th>
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<th>Data</th>
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CONCLUSION

▶ Comparison between AES-PRF and Dual-AES-PRF
  ▶ The security of AES-PRF is **higher** than Dual-AES-PRF from the applicability of differential attacks.
  ▶ Both AES-PRF and Dual-AES-PRF **have only one** round as the security margin.

▶ Choice of the parameter
  ▶ The balanced case AES-PRF\(_{5,5}\) is certainly a natural choice of the design.
  ▶ However, our results indicate that \((s, t) = (4, 6)\) for AES-PRF is potential to be more secure, since the margin with respect to the attacked rounds becomes larger.
Thank you for your attention!

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