Related-Tweak Statistical Saturation Cryptanalysis and Its Application on QARMA

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March 27, 2019 @ Paris

Outline

Motivation and Contributions

Motivation

- Previous statistical saturation attacks are all implemented under single-key setting
- No public attack model under related-key/tweak setting

Contributions

- New cryptanalytic method: related-key/tweak statistical saturation attack
- New distinguishers are conditional equivalent with those utilized in the key/tweak difference invariant bias (KDIB/TDIB) technique
- Automatically search for KDIB/TDIB distinguishers for key-alternating ciphers
- Related-tweak statistical saturation and TDIB attacks on QARMA

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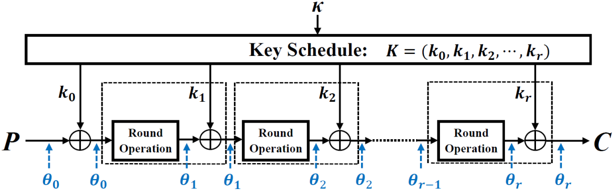
KDIB Technique in Key-Alternating Ciphers

Key-Alternating Block Ciphers [Daemen & Rijmen]



- $\varepsilon_{\theta_{i-1},\theta_i}$: bias of round i
- Bias of θ under κ: $\varepsilon_\theta(\kappa) = 2^{r-1}(-1)^{\theta^t \cdot K} \prod_{i=1}^{r} \varepsilon_{\theta_{i-1},\theta_i}$
- Bias of linear hull (Γ, Λ) under κ:
  $\varepsilon(\kappa) = \sum_{\theta:\theta_0=\Gamma,\theta_r=\Lambda} (-1)^{\theta^t \cdot K} \varepsilon_\theta(0) = \sum_{\theta:\theta_0=\Gamma,\theta_r=\Lambda} (-1)^{d_\theta + \theta^t \cdot K} \varepsilon_\theta$
- $\theta^t \cdot K = \theta^t \cdot K'$ holds for all θ with $\varepsilon_\theta \neq 0$ in the linear hull (Γ, Λ) (*KDIB condition*) $\Rightarrow \varepsilon(\kappa) = \varepsilon(\kappa')$ [Bogdanov *et al.* @ ASIACRYPT'13]

KDIB Technique in Key-Alternating Ciphers

KDIB Distinguisher

- Many linear hulls (Γ, Λ) + a fixed $\Delta \Rightarrow$ KDIB distinguisher, if there exist κ and κ' with $K \oplus K' = \Delta$ satisfying the KDIB condition for each (Γ, Λ)

TDIB Distinguisher

- KDIB attack $\Rightarrow$ TDIB (tweak difference invariant bias) attack, if tweak is alternated
- Tweak has the same effect on the bias of linear hull with key
- $\theta^t \cdot T = \theta^t \cdot T'$ holds for all θ with $\varepsilon_\theta \neq 0$ in the linear hull (Γ, Λ) (*TDIB condition*) $\Rightarrow \varepsilon(t) = \varepsilon(t')$

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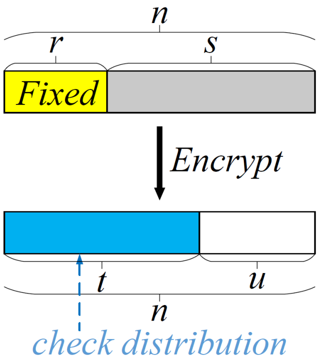
Related-Tweak Statistical Saturation Cryptanalysis

Statistical Saturation Cryptanalysis [Collard & Standaert @ CT-RSA'09]

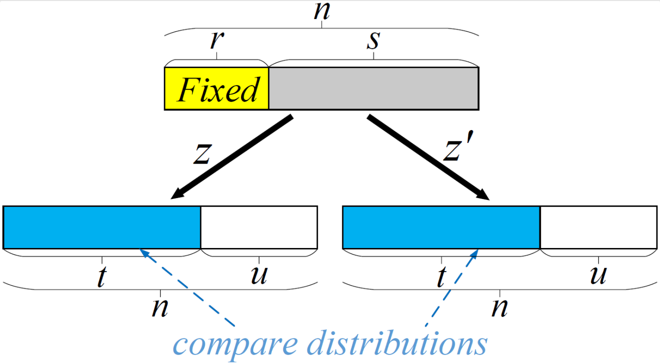
- Fix a part of plaintext bits and take all possible values for the other plaintext bits
- Consider the distribution of a part of the ciphertext value



Related-Tweak Statistical Saturation Cryptanalysis

Related-Key/Tweak Statistical Saturation Cryptanalysis

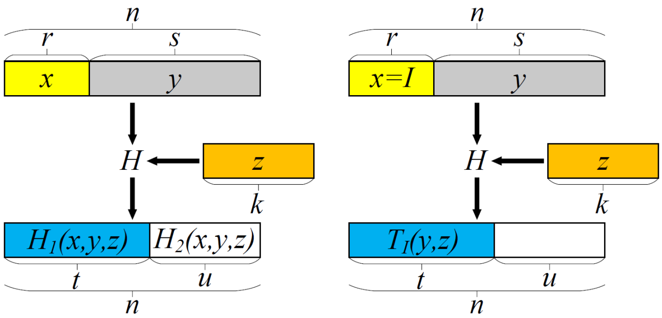
- Fix a part of plaintext bits and take all possible values for the other plaintext bits
- Consider distributions of a part of the ciphertext value under related-key/tweak pairs (z, z'), where $z' = z \oplus \Delta$ and Δ is a fixed value for all possible values of z



Conditional Equivalent Property

Decomposition of the Target Cipher

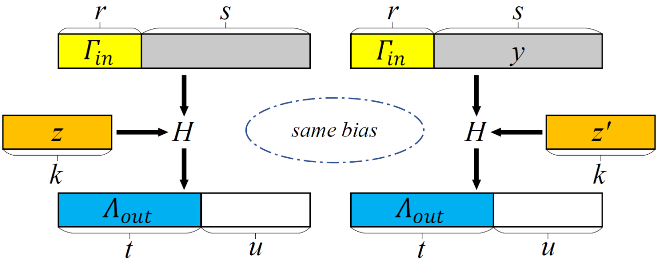
- $H : \mathbb{F}_2^n \times \mathbb{F}_2^k \to \mathbb{F}_2^n$: target cipher with n-bit block and k-bit tweak
- Split the input and output into two parts each:
  $H : \mathbb{F}_2^r \times \mathbb{F}_2^s \times \mathbb{F}_2^k \to \mathbb{F}_2^t \times \mathbb{F}_2^u$, $H(x, y, z) = (H_1(x, y, z), H_2(x, y, z))$
- Define $T_I : \mathbb{F}_2^s \times \mathbb{F}_2^k \to \mathbb{F}_2^t$, $T_I(y, z) = H_1(I, y, z)$



Conditional Equivalent Property

Theorem 1

- (Γ, Λ): the linear hull of H with $\Gamma = (\Gamma_{in}, 0)$ and $\Lambda = (\Lambda_{out}, 0)$, where $\Gamma_{in} \in \mathbb{F}_2^r$ and $\Lambda_{out} \in \mathbb{F}_2^t \backslash \{0\}$
- Given a fixed Δ, we have: the bias is invariant under related-tweak pairs $(z, z' = z \oplus \Delta)$ for all possible mask pairs $(\Gamma_{in}, \Lambda_{out})$ $\Longleftrightarrow$ $T_I(y, z)$ has the same value distribution with $T_I(y, z')$



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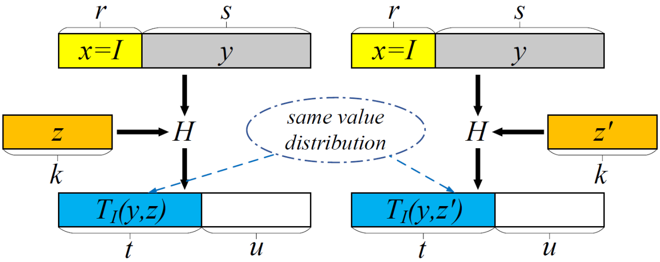
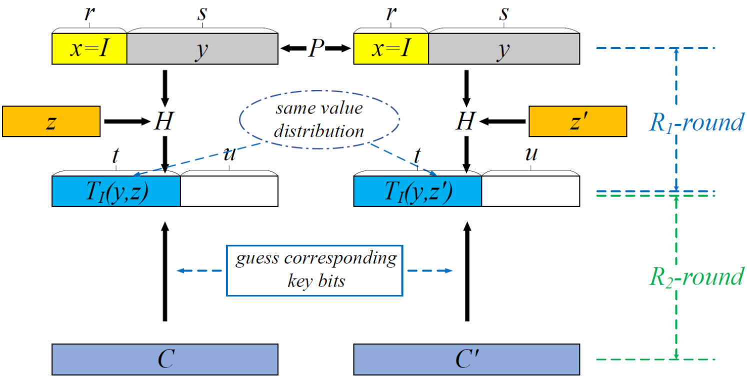


Key Recovery Attack Using Proposed Method

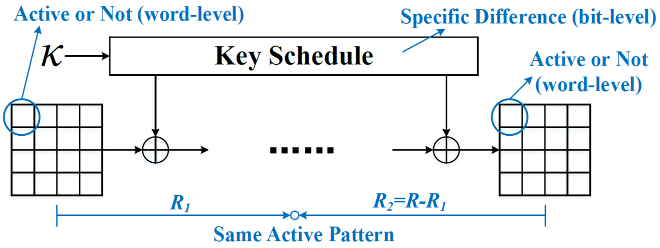


- Reject right key $\alpha_0 = 0$
- Accept wrong key α_1 fulfills
  $\log_2(\alpha_1) \leq (2^t - 1 - t)\, 2^{s+1} - 2^{s(2^t-1)/2}$

Outline

Searching for KDIB Distinguishers with STP

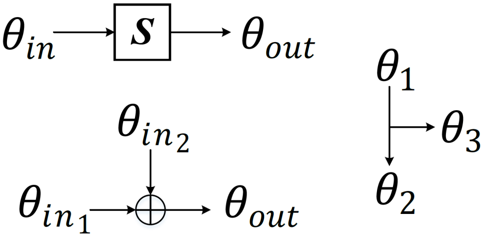
- STP: a decision procedure to confirm if there is a solution to a set of equations
- From previous KDIB attacks (Bogdanov *et al.* @ ASIACRYPT'13), distinguishers were derived at *word-level* for linear masks and *bit-level* for key difference
- Our searching algorithm: *word-level* mask propagation, *bit-level* difference propagation



Searching for KDIB Distinguishers with STP

Part 1. Word-Level Mask Propagation Properties

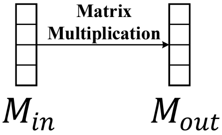
- Substitution: $\theta_{\text{out}} = \theta_{\text{in}}$
- XOR: $\theta_{\text{out}} = \theta_{\text{in}_1} = \theta_{\text{in}_2}$
- Three-Branch: $\theta_3 = 1$, if $\theta_1 = 1$ or $\theta_2 = 1$ holds



Searching for KDIB Distinguishers with STP

Part 1. Word-Level Mask Propagation Properties

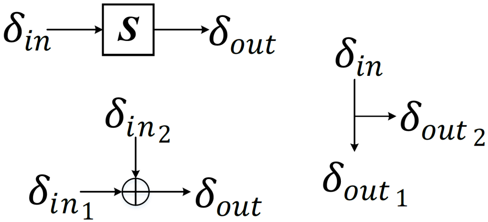
- Deterministic Pattern: M_{out} is unique given M_{in}



- $G = \{M_{in} \mid (M_{in}, M_{out}) \text{ is a deterministic pattern}\}$
- Matrix-Based Linear Layer:
  column-wise active state of input is θ_{in}, column-wise active state of output is θ_{out}. Then $\theta_{out} = M_{out}$ if $\theta_{in} \in G$. Otherwise, $\theta_{out} = (1, 1, 1, 1)^t$

Part 2. Bit-Level Difference Propagation Properties

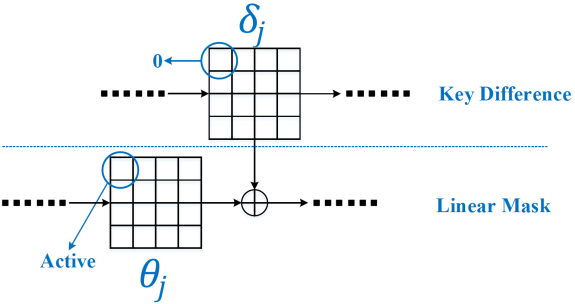
- Substitution: $p = \mathrm{DDT}(\delta_{in}, \delta_{out})$ and $p \neq 0$
- XOR: $\delta_{out} = \delta_{in_1} \oplus \delta_{in_2}$
- Three-Branch: $\delta_{out_1} = \delta_{out_2} = \delta_{in}$



Searching for KDIB Distinguishers with STP

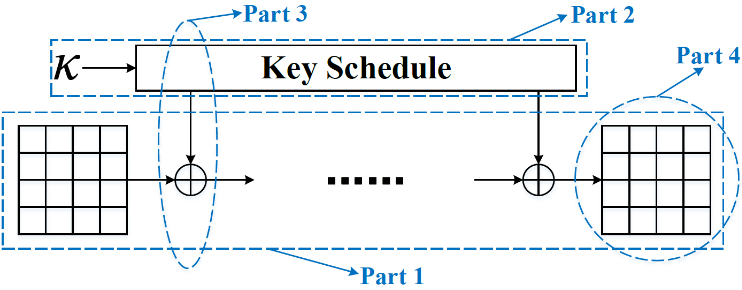
Part 3. Depicting the KDIB Condition

- An r-round linear hull (θ_0, θ_r) and the difference on key $\{\delta_0, \delta_1, \cdots, \delta_r\}$
- KDIB condition: $\oplus_{j=0}^{r} \theta_j \cdot \delta_j = 0$ holds for all possible linear trails $\{\theta_0, \theta_1, \ldots, \theta_r\}$ with $\varepsilon_\theta \neq 0$ in this linear hull
- word-level linear masks $\Rightarrow$ word-level KDIB condition



Part 4. Extra Equations

- At least one round key difference is non-zero $\Rightarrow$ exclude trivial solutions
- Describing the active state of input and output mask
- Restricting the total propagation probabilities, for ciphers containing S-box in their key schedule



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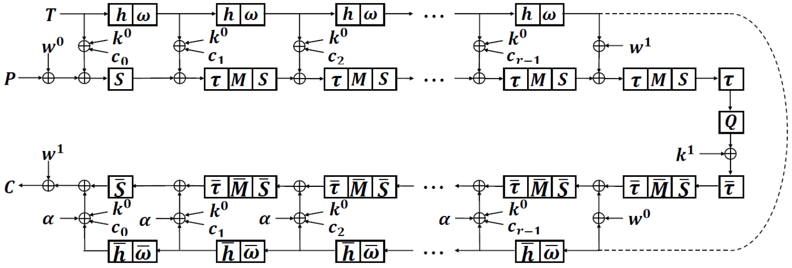
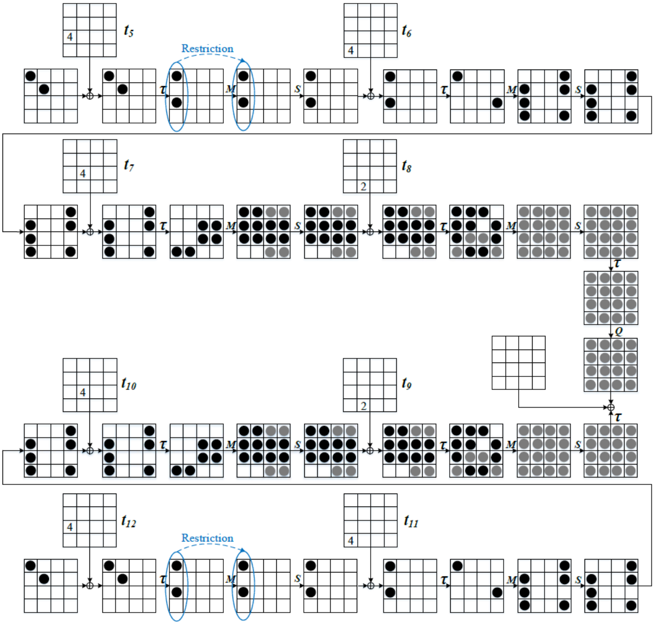
Brief Introduction to QARMA

The Structure of $(2r+2)$-Round QARMA [Avanzi @ ToSC'17]

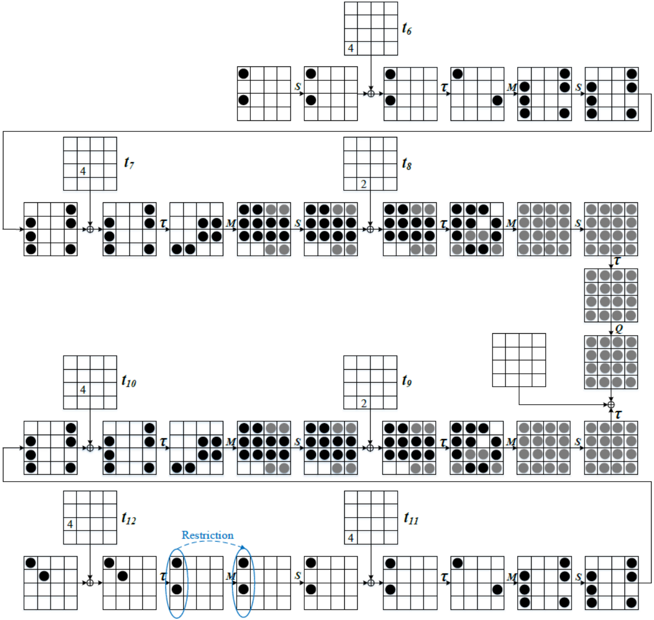


- Two kinds of block sizes: $n = 64$ (QARMA-64), 128 (QARMA-128)
- Key size: $2n$, separated into two parts $w^0 \| k^0$ with same length
- Tweak size: n
- 16 rounds (QARMA-64), 24 rounds (QARMA-128)

One of TDIB Distinguishers for 8-Round QARMA-64



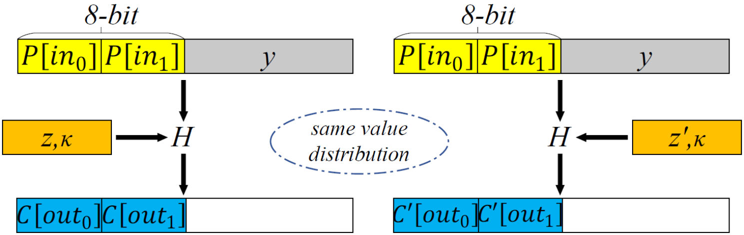
Related-Tweak SS Distinguishers for 8-Round QARMA-64



Convert TDIB into Related-Tweak SS for QARMA-64

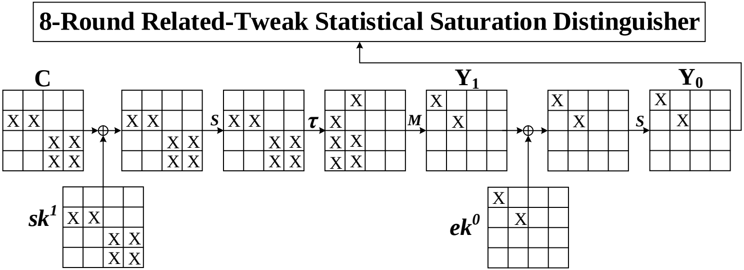
Theorem 3

- (Γ, Λ): linear hull contained in the TDIB distinguishers of the block cipher H
- $\Gamma = (\Gamma[\mathrm{in}_0]||\Gamma[\mathrm{in}_1], 0)$ and $\Lambda = (\Lambda[\mathrm{out}_0]||\Lambda[\mathrm{out}_1], 0)$, where $\Lambda[\mathrm{out}_0] = \Lambda[\mathrm{out}_1]$



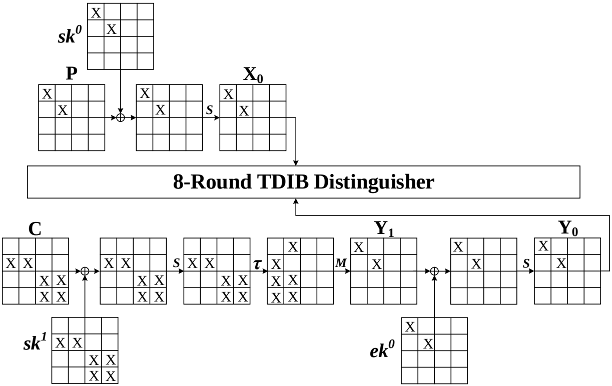
$C[\mathrm{out}_0] \oplus C[\mathrm{out}_1]$ and $C'[\mathrm{out}_0] \oplus C'[\mathrm{out}_1]$: same value distribution

Related-Tweak SS Attacks on 10-Round QARMA-64



Attacks	Rounds	Data	Time	Memory	#tks	Reference
MITM	8	2^{16} CPT	2^{33}	2^{89} 64-bit	1	Li & Jin @ 2018
MITM	9	2^{16} CPT	2^{48}	2^{89} 64-bit	1	Li & Jin @ 2018
RT SS	10	2^{59} CPT	2^{59}	$2^{29.6}$ bits	8	Our Result

TDIB Attacks on 11-Round QARMA-128



Attacks	Rounds	Data	Time	Memory	#tks	Reference
MITM	10	2^{88} CPT	2^{156}	2^{145} 128-bit	1	Li & Jin @ 2018
TDIB	11	$2^{126.1}$ KPT	$2^{126.1}$	2^{71} bits	4	Our Result

Thanks for Your Attention!