On Invariant Attacks

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Outline

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The Real Impact of Lightweight Crypto

Lightweight Crypto

Ligthweight crypto tends to be

- more aggressive
- less standard

Main advantage:

New insights

We learn more about the basics on how (not) to design secure ciphers.

The Real Impact of Lightweight Crypto

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Main advantage:

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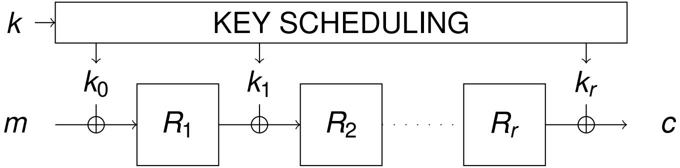
We learn more about the basics on how (not) to design secure ciphers.

It is a pity that NIST states: `[...] submission of algorithms that are not well-understood is discouraged`

Main focus: Key-Alternating Block Cipher



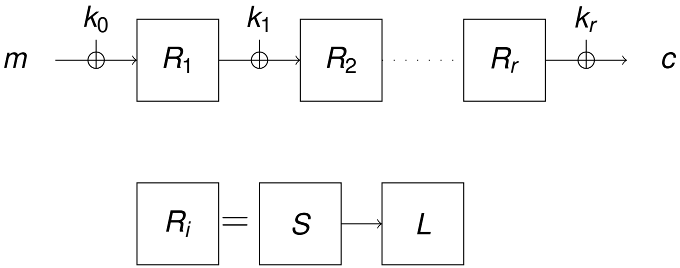
Remark

Most results apply to other structures as well.

Details might change, in particular for

- Partial Non-linear layer
- Cryptographic permutations

Main focus: Key-Alternating Block Cipher

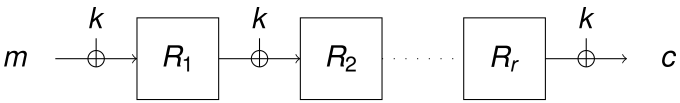


- S: Sboxes
- L: Linear mapping

Minimal Keys-Scheduling

Simplify the Key-Scheduling

- Use the same key in every round
- add round constants



Minimal Keys-Scheduling

Question

Is this a good idea?

- When picking the round constants at random: This is sound.
- Otherwise: Beware of symmetries.

Symmetries

What you do not want (e.g.):

- A symmetric plain-text $p = (x||x)$
- with a symmetric key $k = (y||y)$
- produces always a symmetric cipher-text $c = (z||z)$

One possible abstraction:

Invariant Subspaces

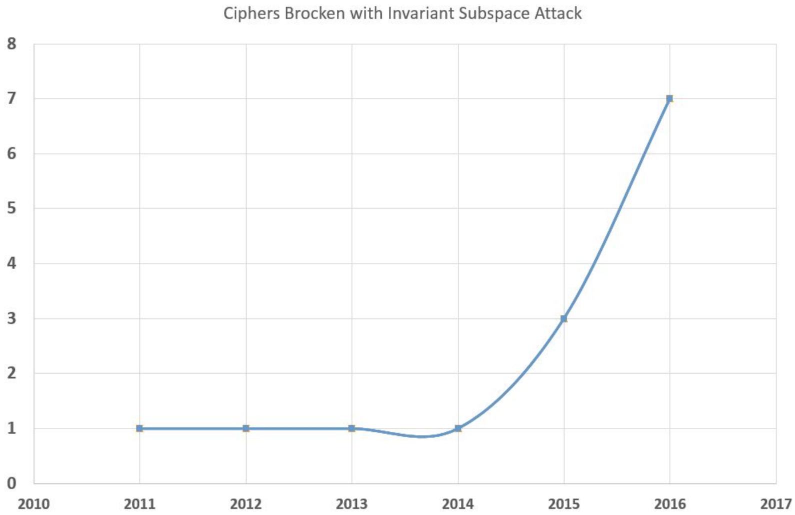
A symmetry is an affine subspace that is (for weak keys) invariant under encryption.

Do those things happen?

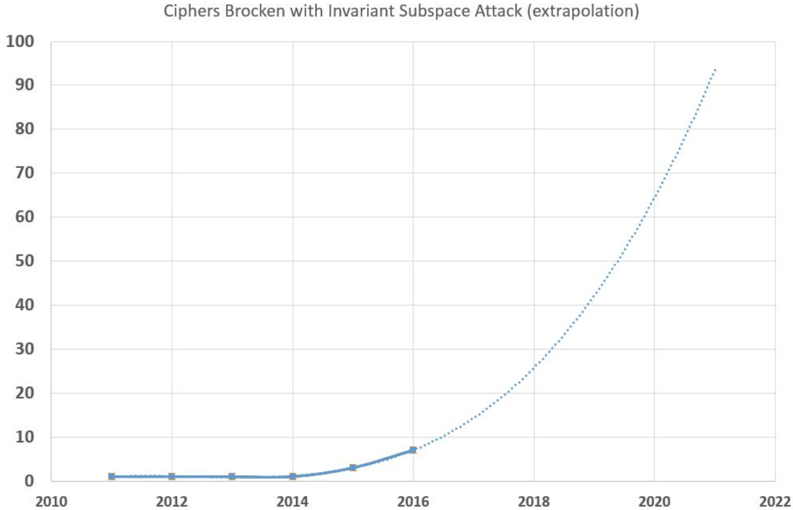
Examples

1. PRINTCipher ('11)
2. iSCREAM ('15)
3. Robin ('15)
4. Zorro ('15)
5. Midori ('16)
6. Haraka (v.0) ('16)
7. Simpira (v.1) ('16)
8. NORX (v 2.0) ('17)

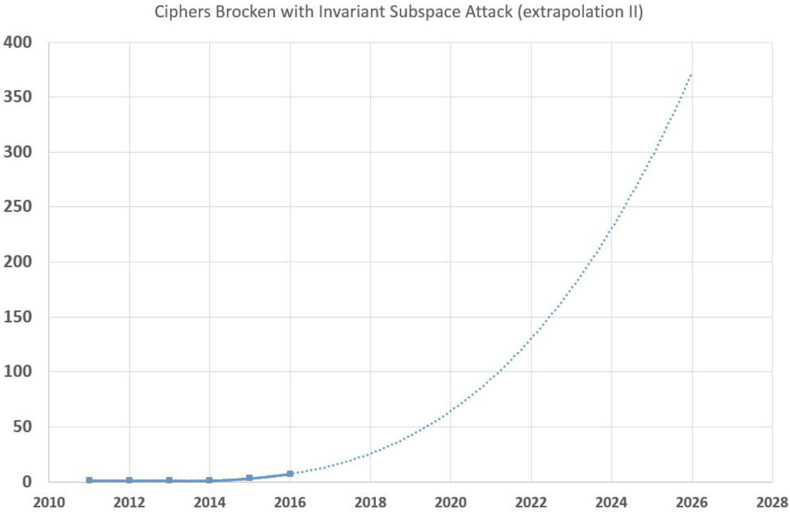
A trend- and were it might lead to (I/III)



A trend- and were it might lead to (II/III)



A trend- and were it might lead to (III/III)



PRIDE v.0

Insider information: I/III

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Email 3 days before the submission deadline

```
Von: Benedikt
An: Mich
Betreff: PRIDE Test Vektoren


Ist das hier ein Grund, sich Sorgen zu machen?
```

- key = 00000000000000000000000000000000
- plaintext = 0000000000000000
- ciphertext = 0000e87b0000eee2

Benedikt

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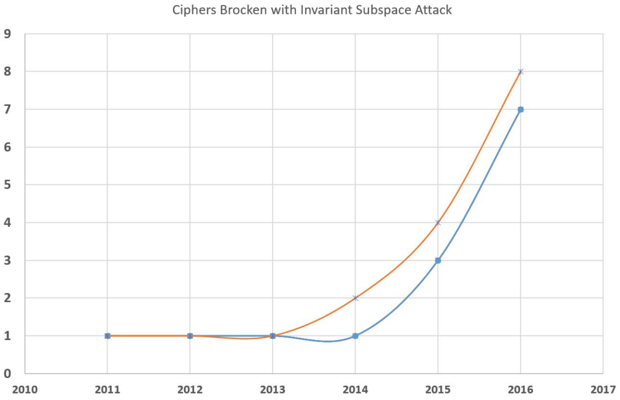
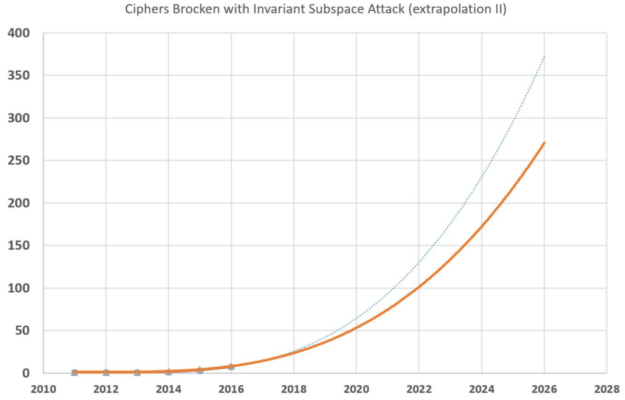
Benedikt

Good for PRIDE, but...

The Impact of Fixing PRIDE

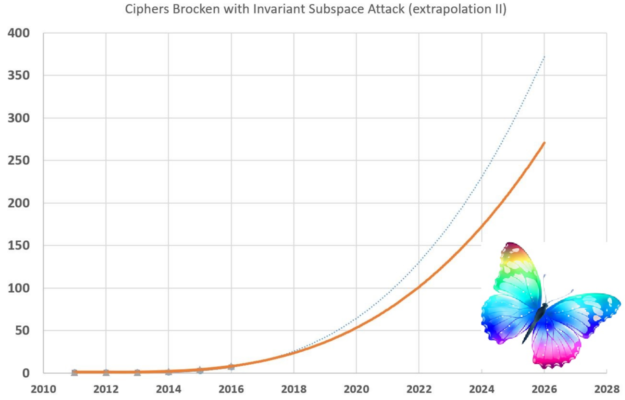


The Impact of Fixing PRIDE



Fixing PRIDE lead to 100 ciphers more being broken in the future

The Impact of Fixing PRIDE



Fixing PRIDE lead to 100 ciphers more being broken in the future

Outline

Origin

First attack with this name:

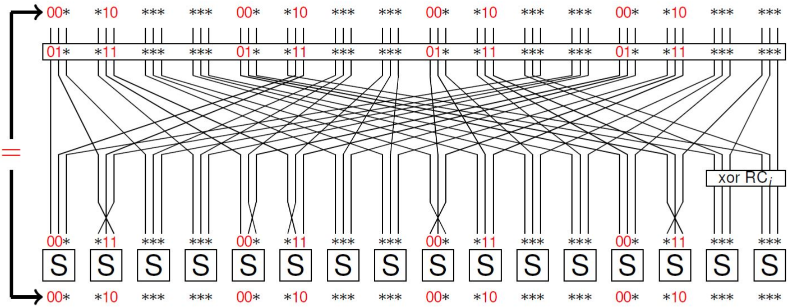
Abdelraheem et al '11

Invariant Subspace Attack on PRINTCIPHER-48.

Several similar ideas previously, in particular

- non-linear approximations
- partitioning cryptanalysis

PRINTCIPHER-48



PRINTCIPHER-48 Attack

Summary

- Prob 1 distinguisher for full cipher
- 2^{50} out of 2^{80} keys weak.
- Similar for PRINTCIPHER-96

Abstraction:

$$F(U \oplus a) = U \oplus b$$

If $k \in U \oplus (a \oplus b)$

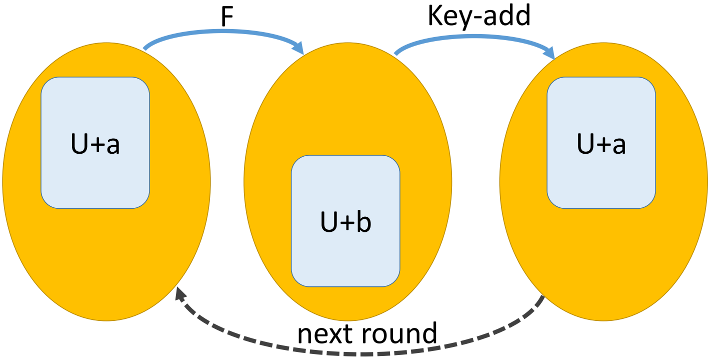
$$F_k(U \oplus a) = U \oplus a$$

Thus an invariant subspace

Question

How to detect it automatically?

The General Idea



- $F(U + a) = U + b$
- $k \in U + (a + b)$ then $U + b + k = U + a$
- Iterative for all rounds (for identical round keys).

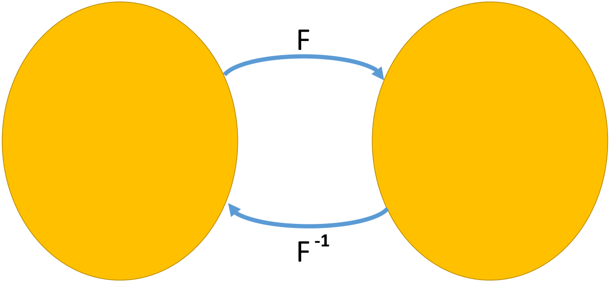
The General Idea

Generic Algorithm (Minaud, Rønjom, L, EC 2015)

Guess a subspace of U. Map it back and forth.

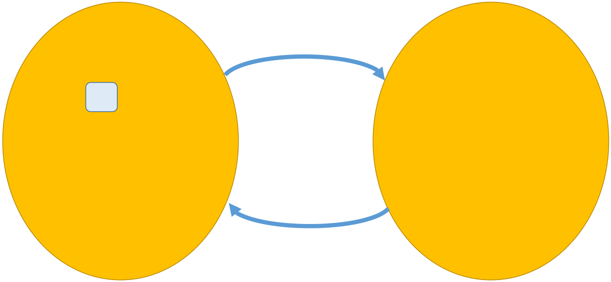
- If the guess was correct: Recovers U
- If not: Find trivial solution.

The General Idea



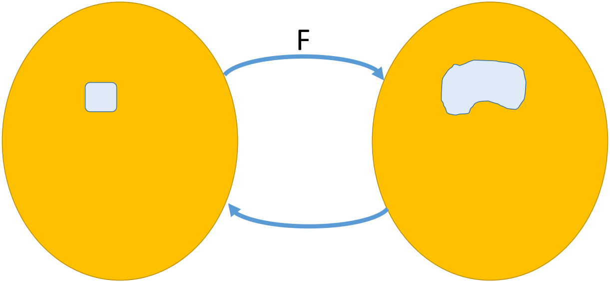
$F := \mathbb{F}_2^n \to \mathbb{F}_2^n$ (permutation)

The General Idea



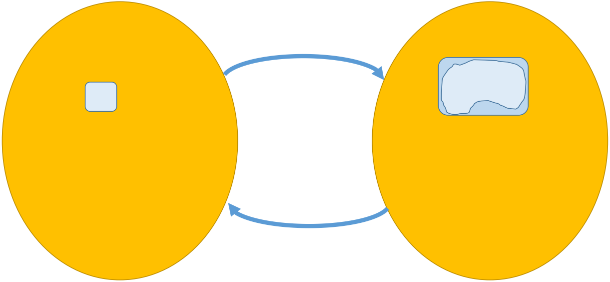
1) Guess a subspace of U

The General Idea



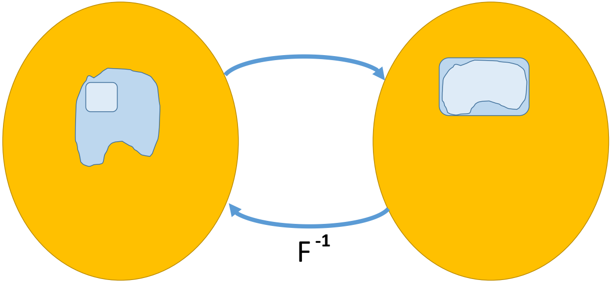
2) Map it using *F*

The General Idea



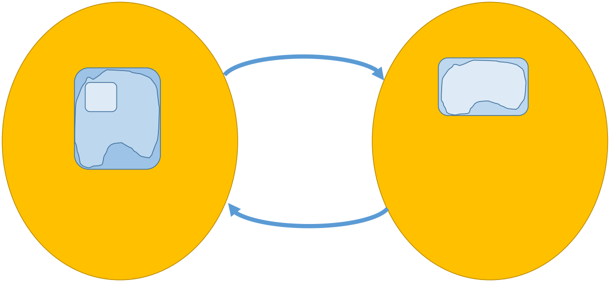
3) Compute the linear span

The General Idea



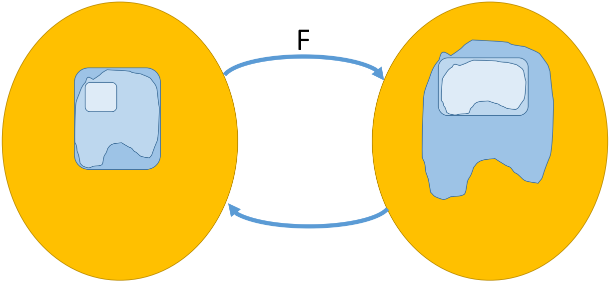
4) Map it using F^{-1}

The General Idea



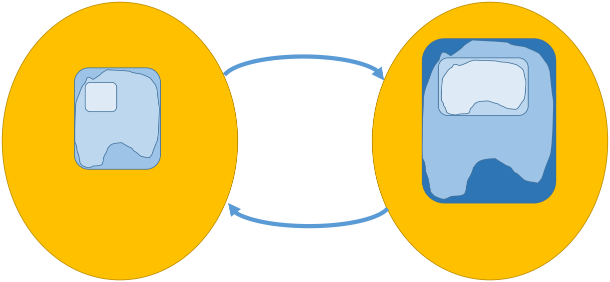
5) Compute the linear span

The General Idea



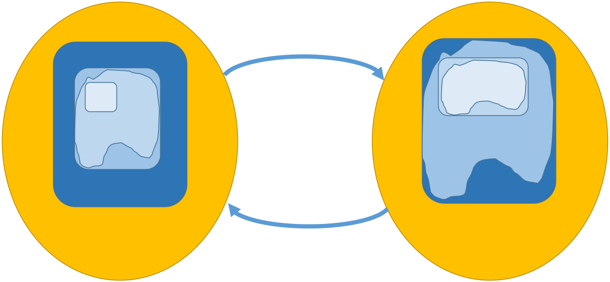
6) Map it using *F*

The General Idea



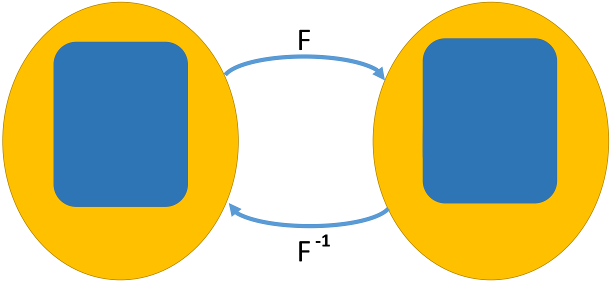
7) Compute the linear span

The General Idea



8) Map it using F^{-1}

The General Idea



9) ...until it stabilizes. Done.

Some Further Considerations

Block length: n

Running Time

Roughly $2^{3(n-d)}$ for the initial guess if an invariant subspace of dim. d exists.

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Block length: n

Running Time

Roughly $2^{3(n-d)}$ for the initial guess if an invariant subspace of dim. d exists.

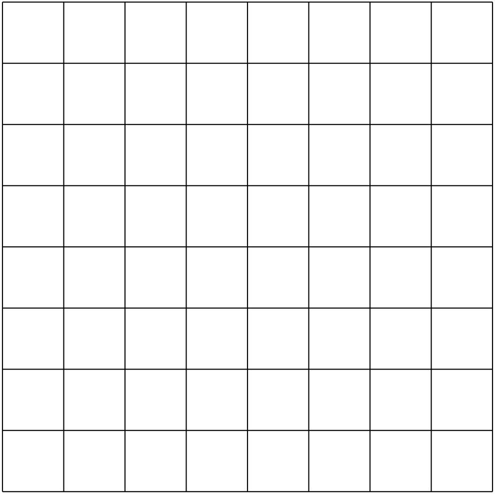
Much better: Include round constants in the initial guess.
Guess only the offset.

Reduced Running Time

2^{n-d} when an invariant subspace of dim. d exists.

Still not satisfactory...

Robin and iScream



One square is a bit. Columns are stored in registers

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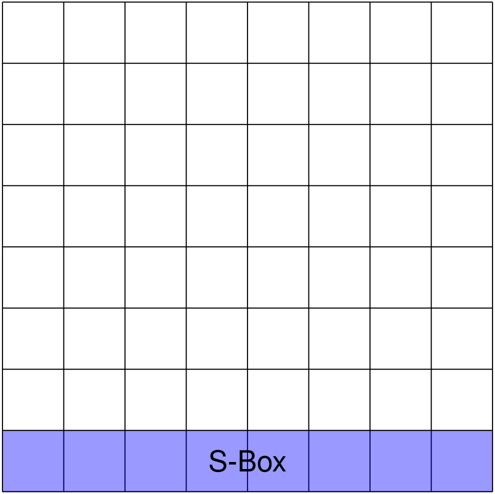

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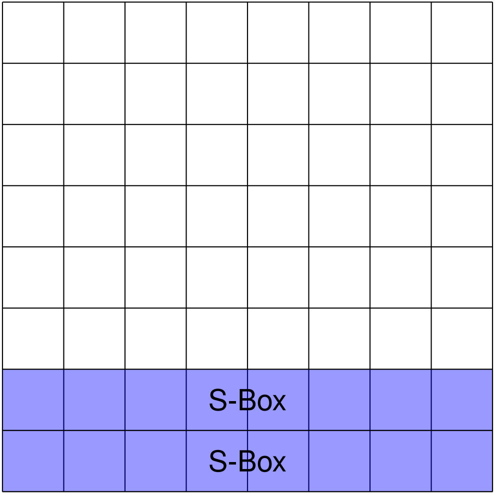

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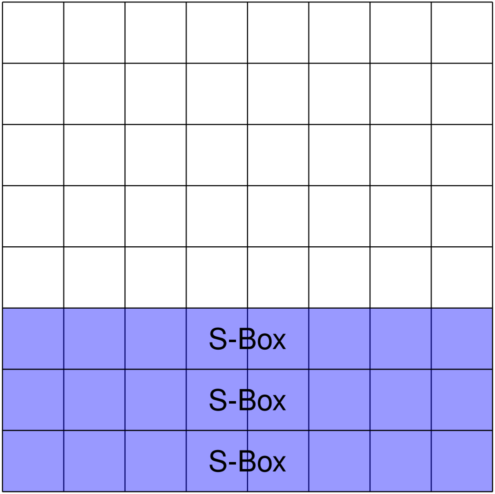

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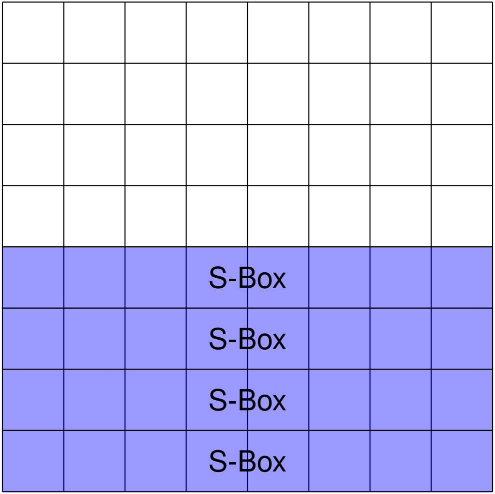

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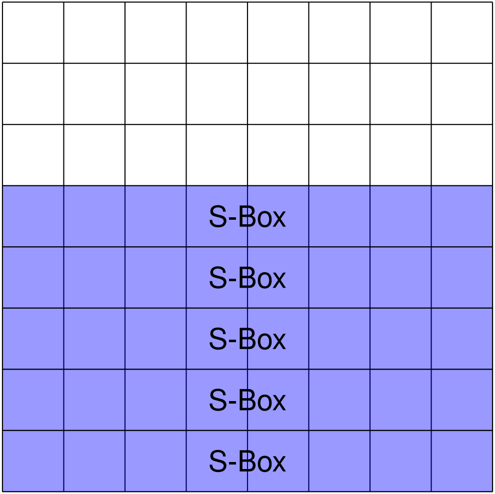

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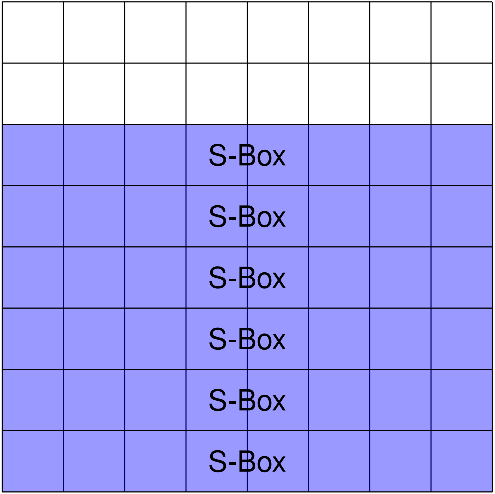

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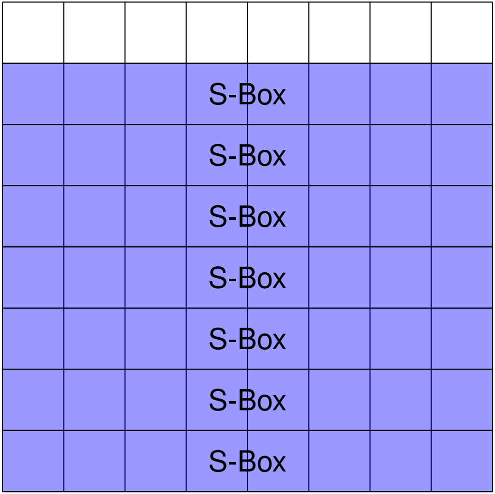

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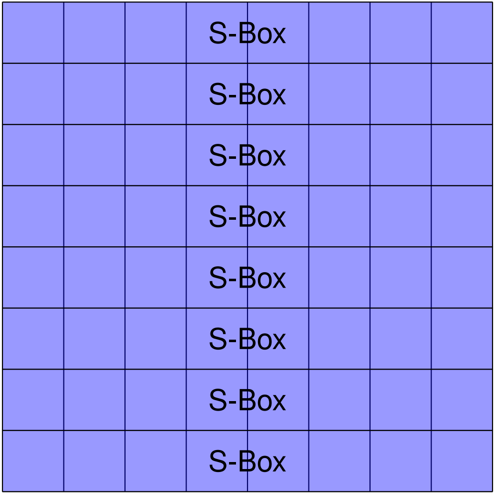

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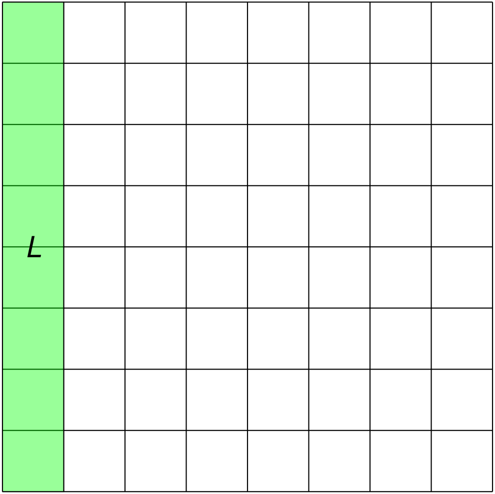

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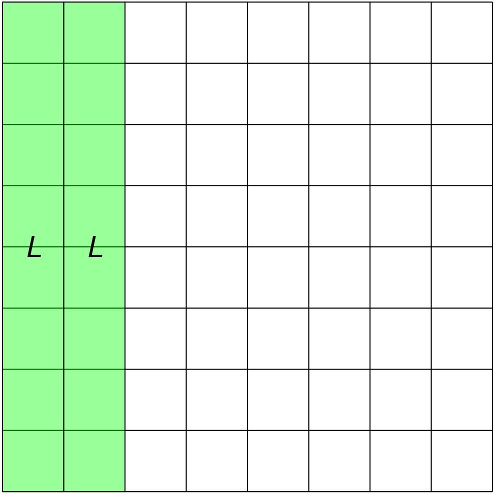

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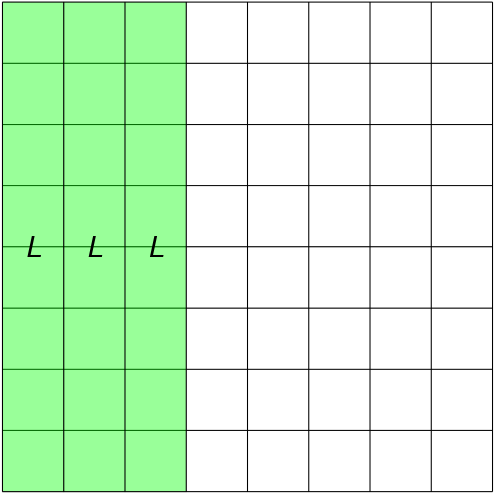

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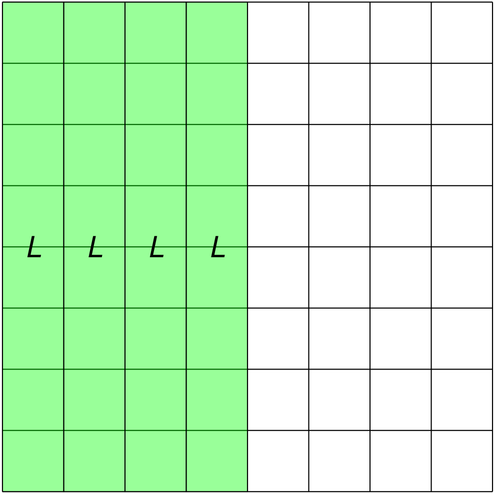

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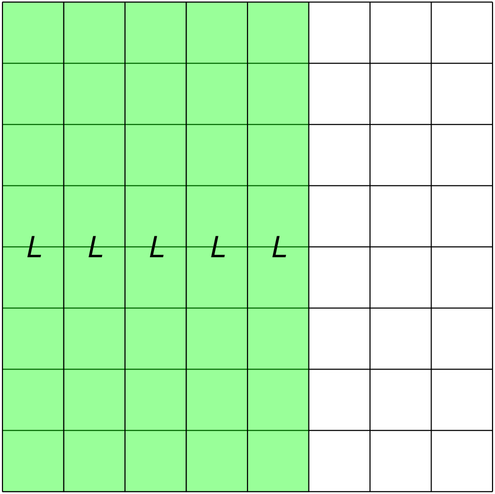

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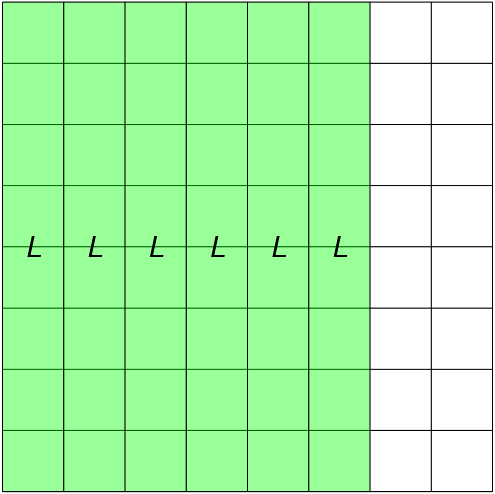

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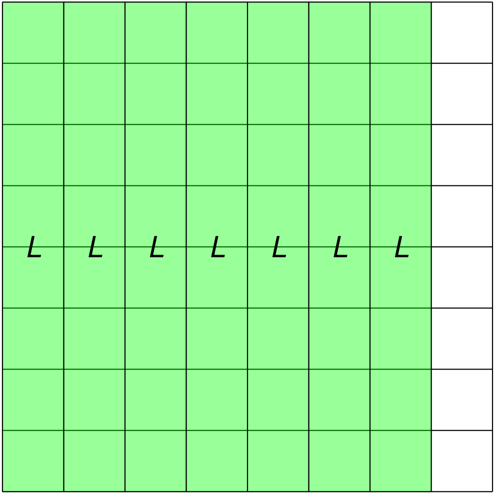

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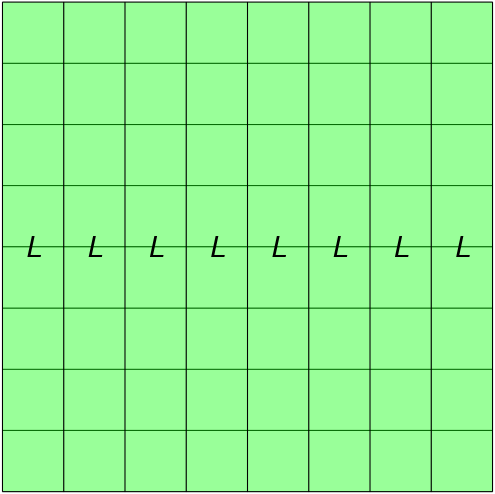

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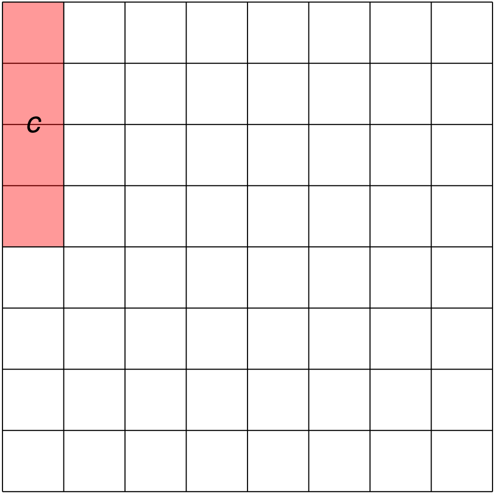

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Applications to Zorro, Robin and iScream

Easy but Powerful

Allows to detect some things

- 32 dim subspace for Robin
- … and for Zorro

Improve Afterwards

The tool detects a (minimal) invariant subspace. Careful analysis increases attack and understanding.

The Robin Sbox

$$00000000 \to 00000000$$
$$10000000 \to 10100001$$
$$01100100 \to 01100100$$
$$11100100 \to 11000101$$
$$00100001 \to 00100001$$
$$10100001 \to 10000000$$
$$01000101 \to 01000101$$
$$11000101 \to 11100100$$

$$S(*, a, b, 0, 0, a, 0, a \oplus b) = (*, \alpha, \beta, 0, 0, \alpha, 0, \alpha \oplus \beta)$$

A Problem of Robin and iScream

*	a_7	b_7	0	0	a_7	0	c_7
*	a_6	b_6	0	0	a_6	0	c_6
*	a_5	b_5	0	0	a_5	0	c_5
*	a_4	b_4	0	0	a_4	0	c_4
*	a_3	b_3	0	0	a_3	0	c_3
*	a_2	b_2	0	0	a_2	0	c_2
*	a_1	b_1	0	0	a_1	0	c_1
*	a_0	b_0	0	0	a_0	0	c_0

$$c_i = a_i \oplus b_i \quad \gamma_i = \alpha_i \oplus \beta_i$$

A Problem of Robin and iScream

*	a_7	b_7	S-Box		a_7	0	c_7
*	a_6	b_6	S-Box		a_6	0	c_6
*	a_5	b_5	S-Box		a_5	0	c_5
*	a_4	b_4	S-Box		a_4	0	c_4
*	a_3	b_3	S-Box		a_3	0	c_3
*	a_2	b_2	S-Box		a_2	0	c_2
*	a_1	b_1	S-Box		a_1	0	c_1
*	a_0	b_0	S-Box		a_0	0	c_0

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*	a_2	b_2	S-Box		a_2	0	c_2
*	a_1	b_1	S-Box		a_1	0	c_1
*	α_0	β_0	0	0	α_0	0	γ_0

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A Problem of Robin and iScream

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*	a_3	b_3	S-Box		a_3	0	c_3
*	a_2	b_2	S-Box		a_2	0	c_2
*	α_1	β_1	0	0	α_1	0	γ_1
*	α_0	β_0	0	0	α_0	0	γ_0

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A Problem of Robin and iScream

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*	α_2	β_2	0	0	α_2	0	γ_2
*	α_1	β_1	0	0	α_1	0	γ_1
*	α_0	β_0	0	0	α_0	0	γ_0

$$c_i = a_i \oplus b_i \quad \gamma_i = \alpha_i \oplus \beta_i$$

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*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
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*	a_5	b_5	0 S-Box	0	a_5	0	c_5
*	α_4	β_4	0	0	α_4	0	γ_4
*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
*	α_1	β_1	0	0	α_1	0	γ_1
*	α_0	β_0	0	0	α_0	0	γ_0

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A Problem of Robin and iScream

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*	α_4	β_4	0	0	α_4	0	γ_4
*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
*	α_1	β_1	0	0	α_1	0	γ_1
*	α_0	β_0	0	0	α_0	0	γ_0

$$c_i = a_i \oplus b_i \quad \gamma_i = \alpha_i \oplus \beta_i$$

A Problem of Robin and iScream

S-Box

*	a_7	b_7	0	0	a_7	0	c_7
*	α_6	β_6	0	0	α_6	0	γ_6
*	α_5	β_5	0	0	α_5	0	γ_5
*	α_4	β_4	0	0	α_4	0	γ_4
*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
*	α_1	β_1	0	0	α_1	0	γ_1
*	α_0	β_0	0	0	α_0	0	γ_0

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*	α_7	β_7	0	0	α_7	0	γ_7
*	α_6	β_6	0	0	α_6	0	γ_6
*	α_5	β_5	0	0	α_5	0	γ_5
*	α_4	β_4	0	0	α_4	0	γ_4
*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
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*	α_5	β_5	0	0	α_5	0	γ_5
*	α_4	β_4	0	0	α_4	0	γ_4
L	L	L	L	L	L	L	L
*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
*	α_1	β_1	0	0	α_1	0	γ_1
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*	α_6	β_6	0	0	α_6	0	γ_6
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	L	L	L	L	L	L	L
*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
*	α_1	β_1	0	0	α_1	0	γ_1
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*	α_6	β_6	0	0	α_6	0	γ_6
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		L	L	L	L	L	L
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				L	L	L	L
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					L	L	L
*	α_3	β_3	0	0	α_3	0	γ_3
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*	α_3	β_3	0	0	α_3	0	γ_3
*	α_2	β_2	0	0	α_2	0	γ_2
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*	α_3	β_3	0	0	α_3	0	γ_3
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c

$$c_i = a_i \oplus b_i \quad \gamma_i = \alpha_i \oplus \beta_i$$

A Problem of Robin and iScream

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$$c_i = a_i \oplus b_i \quad \gamma_i = \alpha_i \oplus \beta_i$$

Generalization

Question

Can we generalize this attack?

Possible directions:

- Not focus on subspaces only
- Statistical Variant
- Allow the subspace to change
- Non-trivial key-scheduling

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- Allow the subspace to change
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Outline

Non-linear Invariant Attacks

- ASIACRYPT 2016
- joint work with Yosuke Todo and Yu Sasaki (NTT)
- Developed not like the storyline suggests.

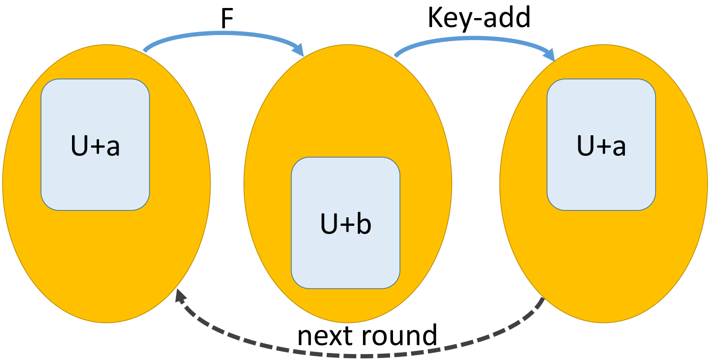
Nonlinear Invariant Attack

Practical Attack on Full **SCREAM**, **iSCREAM**, and **Midori**64

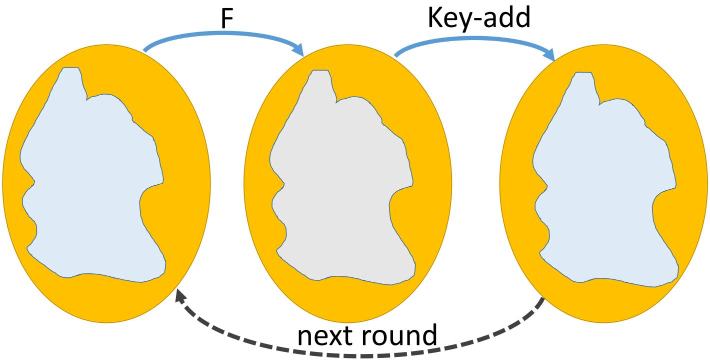
Yosuke Todo and Gregor Leander and Yu Sasaki

Abstract. In this paper we introduce a new type of attack, called *non-linear invariant attack*. As application examples, we present new attacks that are able to distinguish the full versions of the (tweakable) block ciphers Scream, iScream and Midori64 in a weak-key setting. Those attacks require only a handful of plaintext-ciphertext pairs and have minimal computational costs. Moreover, the nonlinear invariant attack on the underlying (tweakable) block cipher can be extended to a ciphertext-

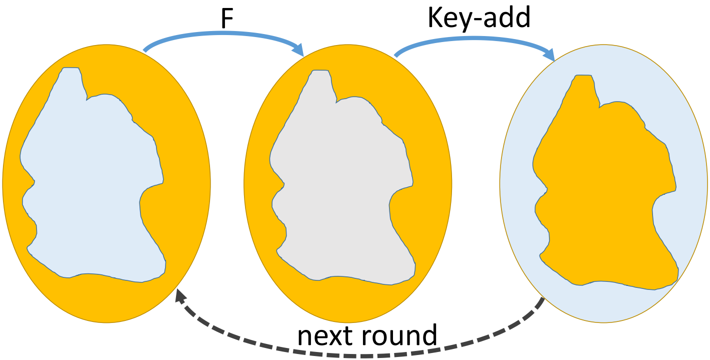
Invariant Subspace Attacks



Nonlinear Invariant Attack (I/II)



Invariant Subspace Attacks (II/II)



Basics

Definition

Given a permutation $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. A Boolean function $g : \mathbb{F}_2^n \to \mathbb{F}_2$ is called a *non linear invariant for F* if

$$g(F(x)) = g(x) + c \quad \forall x$$

where $c \in \mathbb{F}_2$ is a constant.

Link to the picture:

1. Split $\mathbb{F}_2^n$ into two sets

$$\begin{aligned} A &:= \{x \mid g(x) = 1\} \\ B &:= \{x \mid g(x) = 0\} \end{aligned}$$

2. $F(A) = A$ and $F(B) = B$ $(c = 0)$
3. $F(A) = B$ and $F(B) = A$ $(c = 1)$

Applications

Applications

This leads to attacks on

- iSCREAM
- Midori64
- SCREAM (v.3)

Can be extended to a cipher-text only attack

- when used in certain modes (e.g. CBC, CTR) mode
- same message encrypted multiple times

with very low complexity.

Results

	weak keys	recovered bits	data	time
SCREAM (v.3)	2^{96}	1/4	33 CT	32^3
iSCREAM	2^{96}	1/4	33 CT	32^3
Midori64	2^{64}	1/2	33 CT	32^3

More details in the paper. In particular

- The details
- An explanation why that attack works on those ciphers

How it was actually developed

Insider information II/III: How it was actually developed.

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Yosuke Todo was visiting RUB

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Yosuke Todo was visiting RUB

Division Property

A set $\mathbb{X}$ has division property $\mathcal{D}_k^n$ if

$$\sum_{x \in \mathbb{X}} x^u = 0$$

for all $u \in \mathbb{F}_2^n$ with $\mathrm{wt}(u) < k$.

$$\Leftrightarrow$$

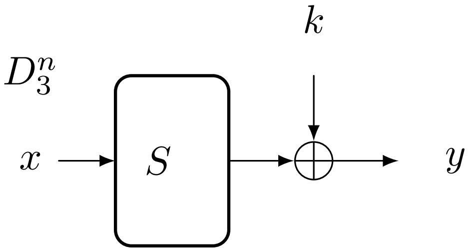
For all $f : \mathbb{F}_2^n \to \mathbb{F}_2$ with $\deg(f) < k$ we have

$$\sum_{x \in \mathbb{X}} f(x) = 0$$

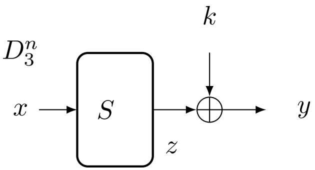
How it was actually developed

Research Question

Can we overcome one Sbox without guessing the entire key?



How it was actually developed



Find a function

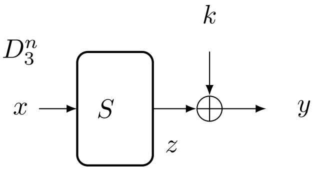
$$g : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$$
$$z \mapsto g(z)$$

1. $g(z)$ does not depend non-linear on all bits of z.
2. Equals a quadratic function f in the inputs x

That is:

$$g(z) = g(S(x)) = f(x)$$

How it was actually developed



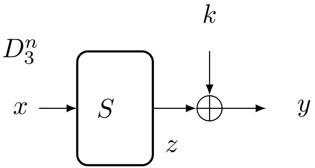
$$f(x) = g(z)$$

Attack Outline

- Guess parts of the key
- Compute $g(z)$
- For correct key we get

$$\sum_z g(z) = \sum_{x \in \mathbb{X}} f(x) = 0$$

How it was actually developed



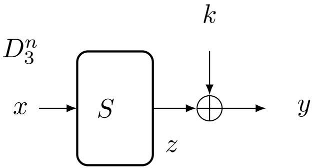
Looking at many examples we found:

Scream

$$x_1 x_2 + x_0 + x_2 + x_5 = z_1 z_2 + z_0 + z_2 + z_5 + 1$$

That is $f = g + 1$.

How it was actually developed



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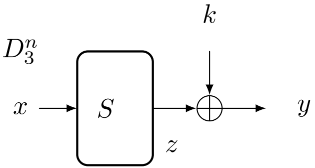

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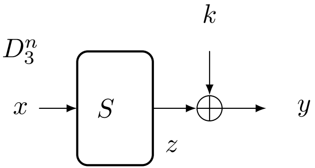

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$$x_1 x_2 + x_0 + x_2 + x_5 = z_1 z_2 + z_0 + z_2 + z_5 + 1$$

That is $f = g + 1$.

- interesting...
- just a coincidence?
- can we do anything with that?

How it was actually developed

One Month Later: Email from Yosuke

```
I have new results, and I want to submit this
result to Asiacrypt 2016.
```

Outline

Avoiding those Attacks

Proving Resistance against Invariant Attacks: How to Choose the Round Constants

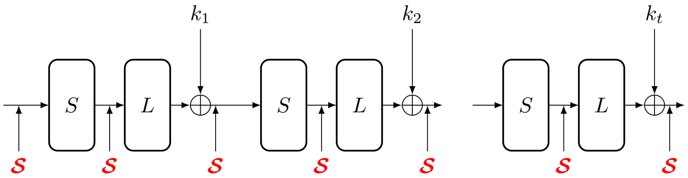
Christof Beierle[1], Anne Canteaut[2], Gregor Leander[1], and Yann Rotella[2]

[1] Horst Görtz Institute for IT Security, Ruhr-Universität Bochum, Germany
{christof.beierle, gregor.leander}@rub.de
[2] Inria, Paris, France
{anne.canteaut, yann.rotella}@inria.fr

Abstract. Many lightweight block ciphers apply a very simple key schedule in which the round keys only differ by addition of a round-specific constant. Generally, there is not much theory on how to choose appropriate constants. In fact, several of those schemes were recently broken using invariant attacks, i.e. invariant subspace or nonlinear invariant attacks. This work analyzes the resistance of such ciphers against invariant attacks and reveals the precise mathematical properties that render those attacks applicable. As a first practical consequence, we prove that some ciphers including Prince, Skinny-64 and Mantis_7 are not vulnerable to invariant attacks. Also, we show that the invariant factors of the linear

A satisfactory(?) answer for the designers

Invariants under L and S



Focus on invariants that are

- Invariant for S-Layer
- Invariant for all $\text{Add}_{k_i} \circ L$

Not much of a restriction!?

Most known attacks are of this form.

Exception: ASIACRYPT 2018

Implication

$$g(L(x)+k_i) = g(x)+\varepsilon_i \text{ and } g(L(x)+k_j) = g(x)+\varepsilon_j$$
$$\Rightarrow g(L(x)+k_i) = g(L(x)+k_j) + (\varepsilon_i+\varepsilon_j)$$
$$\Leftrightarrow g(y+k_i+k_j) = g(y) + (\varepsilon_i+\varepsilon_j)$$

Linear Structure

$(k_i + k_j)$ is a linear structure of g.

Recall:

Linear space of a Boolean function g

$$\mathrm{LS}(g) := \{\alpha \in \mathbb{F}_2^n : x \mapsto g(x+\alpha) + g(x) \text{ is constant}\}$$

More Implications

Lemma

Let g be an invariant

- *for S-Layer*
- *for all* $Add_{k_i} \circ L$

then

- $\mathrm{LS}(g)$ *contains* $k_i + k_j$
- $\mathrm{LS}(g)$ *is invariant under L.*

Focus on the simplest key-scheduling:

$$k_i = k + c_i$$

That is

$$k_i + k_j = c_i + c_j$$

Existence of Non-Trivial Non-linear Invariant

Given

$$D := \{(c_i + c_j) \mid i, j \in \{1, \ldots, r\}\}$$

we define

$$W_L(D) := \text{smallest L-invariant subspace containing } D$$

Question

Is there a non-trivial invariant g for the S-Layer such that

$$W_L(D) \subseteq \mathrm{LS}(g)?$$

Dimension of $W_L(D)$

Corollary

If $\dim(W_L(D)) \geq n - 1$ *than such a g does not exist.*

Proof.

Otherwise S-Layer has linear component. □

Proves that the attack does not work for e.g.

- LED
- Skinny-64-64

More General

Theorem

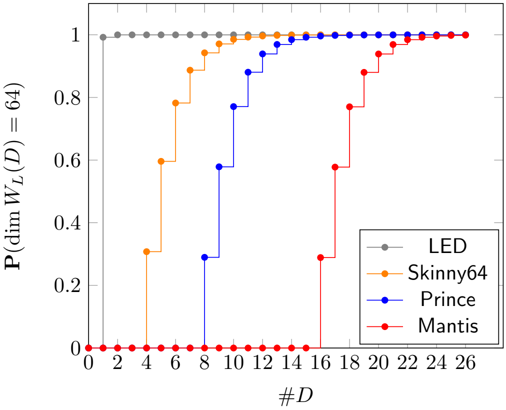
Let $Q_1, \ldots Q_r$ be the invariant factors of L. For any $t \leq r$

$$\max_{c_1,\ldots,c_t} \dim W_L(\{c_1, \ldots, c_t\}) = \sum_{i=1}^{t} \deg Q_i$$

Study the invariant factors of the linear layer!

- Explains required number of constants
- Explains how to choose them
- Works independent of S-layer.

Examples


$\mathbf{P}(\dim W_L(D) = 64)$
0
0.2
0.4
0.6
0.8
1
0 2 4 6 8 10 12 14 16 18 20 22 24 26
$\#D$
LED
Skinny64
Prince
Mantis

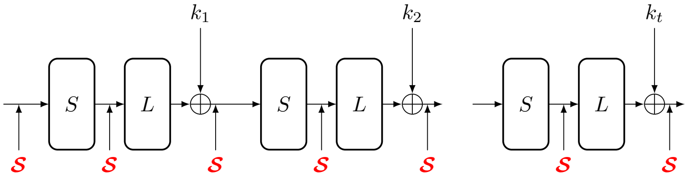
But....

Insider Information III/III

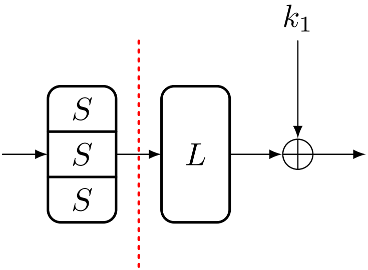
But....

Insider Information III/III
Remember:

- It has to work for both S and L
- analysis independend of the Sbox



Cosmetic Changes



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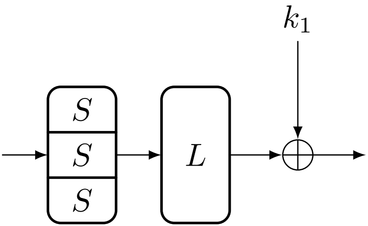

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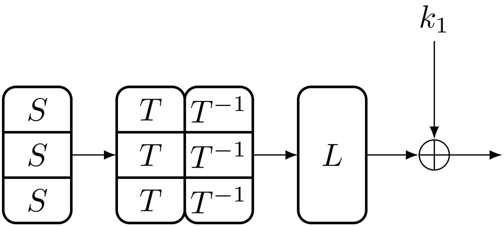

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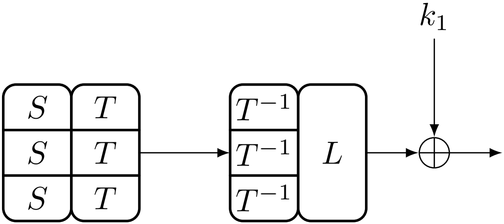
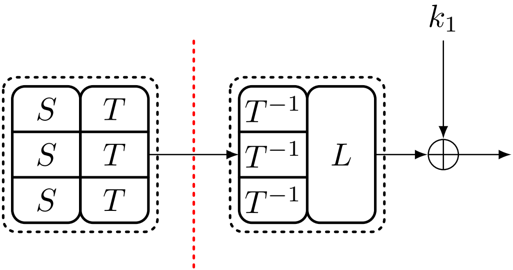
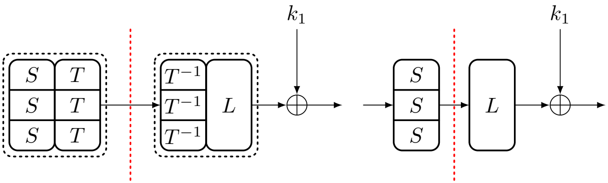


Cosmetic Changes



What does it mean?



The argument might

- work for one
- but not for the other

representation!

What does it mean?

Important Restriction

The argument is an argument for the security of a representation of the cipher

- Not what we really want
- Can we remove the restriction?

More General

Not an uncommon restriction.

What does it mean?

Important Restriction

The argument is an argument for the security of a representation of the cipher

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Not an uncommon restriction.

Thank you very much for your attention!